The spin-wave modes excited in the in-plane-magnetized YIG disk are calculated as shown in Fig. S1 and Table S1 (Refs. 1–5). The dispersion relations for the case of backward volume modes (BVMs) have been calculated for different thickness modes $n$ using the simplified analytical equations in Ref. 1 (see Fig. S1b). For the limit of $k \to 0$, all the dispersion characteristics tend to converge to the ferromagnetic resonance (FMR) frequency. The FMR frequency $f_{\text{FMR}} = 3.28$ GHz is calculated by using
\[ f_{FMR} = \frac{\gamma}{2\pi} \sqrt{H(H + 4\pi M_s - H_d)} \]

(S1)

where \( \gamma = 1.76 \times 10^7 \text{ (Oe s)}^{-1} \), \( H = 630 \text{ Oe} \), \( 4\pi M_s = 1823 \text{ G} \), and \( H_d = 270 \text{ Oe} \) are the gyromagnetic ratio, the static magnetic field, the saturation magnetization, and the demagnetization field, respectively. Considering that spin-wave modes can only be excited in a certain range of wavenumbers given by the width of the launching microstrip line, the higher order thickness modes do not play an important role in the spin-wave spectrum and their frequencies are all localized close to the FMR frequency. Only the lowest BVM \( n = 0 \) is responsible for the signal observed at frequencies below the FMR frequency (see Fig. S1a). In a similar manner, we calculated the dispersion relations for the Damon Eshbach mode (DEM)-free and DEM-waveguide YIG surfaces (see Fig. S1b); (the bottom surface of the YIG disk is close to the microwave antenna).

To calculate the spin-wave mode wavenumbers, we used the equations introduced by Zivieri (Ref. 2; equation (45) for BVMs and equation (49) for DEMs). The wavenumbers of the symmetric (S) and antisymmetric (AS) spin-wave modes are shown in Table S1. These wavenumbers were used to calculate the frequencies of the DEMs, such that the wavenumbers were incorporated into the calculated dispersion relations and the resulting frequencies were compared with the measured spectrum (Ref. 3). Theoretical and experimental results are in good agreement (see Fig. S1a). The DEMs with frequencies larger than 4 GHz are associated with travelling MSSWs (DEM-waveguide surface). The modes with frequencies below the FMR frequency are identified as BVMs. Nevertheless, taking a high density of these modes and a comparatively large FMR line width of the polycrystalline YIG sample into account, it is obvious that these modes
cannot be separated in the experiment and are only visible as a single wide absorption band (see Fig. S1).

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Spin-wave wavenumber $k$ (rad/cm)</th>
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<tbody>
<tr>
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<tr>
<td></td>
<td>S</td>
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</table>

Table S1: Calculated wavenumbers of DEMs and BVMs in in-plane magnetized disk.

B. Spin-wave mode excitation for thin $Y_3Fe_5O_{12}$ film by narrow antenna
Figure S2: Spin-waves excited in thin single-crystalline $\text{Y}_3\text{Fe}_5\text{O}_{12}$ film. a, Absorption $S_{11}$ and transmission $S_{21}$ spectra for 30 $\mu$m-thick $\text{Y}_3\text{Fe}_5\text{O}_{12}$ film by using narrow antennas. b, Calculated frequencies and their dispersions for spin-waves: BVM, FMR, DEM.

In the experimental setup of a long thin-film $\text{Y}_3\text{Fe}_5\text{O}_{12}$ waveguide with 50 $\mu$m-diameter narrow antenna shown in Fig. 4, the spin-wave excitation characteristics are different compared to the disk case shown in Fig. 2. In thin $\text{Y}_3\text{Fe}_5\text{O}_{12}$ films DEMs can be excited in the range of frequencies from the FMR frequency $f_1$

$$f_1 = \frac{\gamma}{2\pi} \sqrt{H(H + 4\pi M_s)}$$  \hspace{1cm} (S2)

up to the maximum frequency of the dipolar DEMs $f_2$

$$f_2 = \frac{\gamma}{2\pi} \left[H + \frac{1}{2}(4\pi M_s)\right]$$ \hspace{1cm} (S3)

where $\gamma = 1.76 \cdot 10^7 \text{ (Oe s)}^{-1}$, $H = 1750 \text{ Oe}$, and $4\pi M_s$ are the gyromagnetic ratio, the static magnetic field, and the saturation magnetization, respectively which can be found in the review paper (Ref. 6). From the experimentally obtained spectra in Fig. 2a and the spatial temperature distribution observed by using an IR camera, the peak and dip at frequency of 6.98 GHz is confirmed as FMR signal. Then, the saturation magnetization is estimated as $4\pi M_s = 1800 \text{ G}$ from equation (S2), and $f_2$ is calculated with these parameters as $f_2 = 7.42 \text{ GHz}$ (Fig. S2b). In the experimental setup used in Fig. S2b, where propagating spin waves (rather than standing spin wave modes) are excited by the narrow antenna, the excitation efficiency of spin waves has to be taken into account (Ref. 7). Because of the limited spin-wave wavenumber excited by the narrow antenna, the excitation of DEMs is observed at a maximum frequency of $f = 7.25 \text{ GHz}$, which is smaller than
We note that the observed oscillations in the DEM transmission signal is due to a beating between the traveling spin-wave signal and the comparably small electromagnetic leakage (which is due to the direct microwave signal transmitted through the air or the dielectric substrate).

C. Numerical calculation of the spatial distribution of surface mode spin waves

To calculate the magnetization dynamics, we numerically solve the Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{\partial}{\partial t} \mathbf{M}(r, t) = -\gamma \mathbf{M}(r, t) \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M}(r) \times \frac{\partial}{\partial t} \mathbf{M}(r)$$

(S4)

by using the object-oriented micromagnetic framework (OOMMF) extended to the 4th-order Runge-Kutta method. Here, $\gamma = 1.76 \times 10^7$ (Oe s)$^{-1}$ is the gyromagnetic ratio and $4\pi M_s = 1823$ G is the saturation magnetization. $H_{\text{eff}}$ is the effective magnetic field that acts on the magnetization. $H_{\text{eff}}$ includes the external applied, exchange, and demagnetization fields. We apply the DC magnetic field $H_{dc} = 630$ Oe in the $z$ direction and the AC magnetic field $H_{ac}$ in the $x$ direction. The demagnetization field is calculated by the convolution of a kernel describing the dipole-dipole interaction between unit cells. The convolution is performed by using the fast Fourier transform technique. We also apply the random magnetic field due to the thermal effect (Ref. 8). We use the damping parameter $\alpha = 0.009$ that is estimated from the FMR spectrum. To calculate the DEM waves in Y$_3$Fe$_5$O$_{12}$ thin film, we perform the simulation of the Y$_3$Fe$_5$O$_{12}$ rectangular thin film by assuming a sample with dimensions of $1000 \ \mu m \times 100 \ \mu m \times 1000 \ \mu m$; the unit cell is $0.1 \ \mu m \times 0.1 \ \mu m \times 1000 \ \mu m$. Since we have not divided the system in the $z$ direction, the model cannot describe
the backward volume modes (BVMs) that have a wave vector parallel to the DC magnetic field. We show snapshots of the transverse component of the magnetization ($M_x$) in Fig. S3 when the FMR (Fig. S3a) and the DEM (Fig. S3b) are excited.

We apply AC magnetic fields with frequencies of 3.3 GHz and 3.6 GHz that correspond to the resonant frequencies of the FMR and the DEM mode, respectively. For modeling the waveguide located under the sample, the amplitude of the AC magnetic field is given by $H_{ac} = H_x \exp(-y\lambda^{-1})$, where $H_x = 1.0$ Oe and $\lambda = 50 \mu$m. One can see in Fig. S3 that the amplitude of the magnetization is uniform for the FMR resonance in the $x$ direction, while it is localized at the end of the sample for the DEM resonance in the $x$ direction.

Figure S3: Snapshots of transverse component of magnetization $M_x$. The frequency of the AC magnetic field is set to a 3.3 GHz and b 3.6 GHz, which correspond to the excitation conditions of the FMR and the DEM, respectively.
D. Observation of nonreciprocal microwave transmission signal induced by surface spin-wave excitation

Wires acting as microwave antennas (antennas 1 and 2) were placed below the Y₃Fe₅O₁₂ sample, and the in-plane magnetic field was applied perpendicular (\(H_1\)) and parallel (\(H_2\)) to the antennas where the magnetostatic spin waves BVMs and DEMs are effectively excited, respectively. The transmitted spectra \(S_{21}\) (solid line) and \(S_{12}\) (dotted line) were measured by sweeping the microwave frequency: 2.5 – 4.5 GHz with the magnetic field \(H = 600\) Oe and the microwave power 0.3 mW. In the geometry of the BVM excitation (BVMs (2.5 – 3 GHz) and the FMR (3.3 GHz)) (Fig. S4a), the observed spectra \(S_{21}\) and \(S_{12}\) are identical. In contrast, in the geometry of DEM excitation (DEM (3.6 – 4.2 GHz) and FMR (3.3 GHz)) (Fig. S3b), the observed intensity of \(S_{21}\) from the DEMs is stronger than that of \(S_{12}\), indicating the nonreciprocity of the DEM of spin waves.

Figure S4: Microwave transmission spectra, transmitted from antenna 1 to antenna 2 (\(S_{21}\)) and vice versa (\(S_{12}\)), are shown for sweeping microwave frequency (\(P = 0.3\) mW) of 2.5 - 4.5 GHz and applying static magnetic field \(H = 600\) Oe perpendicular (\(H_1\)) a and parallel (\(H_2\)) b to the antenna.
E. References


